Drill, Examinations and the Learning of Mathematics

Ken Clements
Universiti Brunei Darussalam
<clements@shbie.ubd.edu.bn>

This paper presents an overview of the findings of three masters-level research investigations that the writer supervised during the period 1998–2001. Each investigation was based on a "multiple perspectives" design, and analyses revealed that most of the Bruneian secondary school children who were taught mathematics in the traditional "skill-drill" way did not learn much mathematics. Data from the studies suggest that emphasis on preparation for forthcoming tests and examinations does not result in students achieving a genuine understanding of mathematical topics. Lim's (2000) summary of a didactical contract between Bruneian secondary mathematics teachers and students is presented. It is argued that the teachers and the students, and indeed whole school communities, are continuing to exercise uninformed faith in a method of teaching and learning which is not "producing the goods."

Schools in Brunei Darussalam place great emphasis on the preparation of students for forthcoming tests and examinations (Khoo, 2001; Lim, 2000; Noridah, 1999). Primary school students are prepared for the high-stakes Primary Certificate of Education (PCE) examinations, and mean PCE performances of schools are ranked and monitored. Similarly, lower secondary students are prepared for the end-of-Form 3 *Penilaian Menengah Bawah* PMB national examinations, and upper-secondary students are prepared for the University of Cambridge Local Examination Syndicate's (UCLES's) O- and A-level examinations. There is also an N-level examination, which is organised within Brunei Darussalam for those not considered to be ready for O-level studies. All mathematics teachers are expected to test their classes regularly (at least once a month), and special lessons are dedicated to revision, tests, and to giving overviews of test results.

The Multiple Perspectives Approach to Researching the Teaching and Learning of Mathematics in Schools in Brunei Darussalam

Mathematics Classroom Patterns in Brunei Darussalam: Lim's (2000) Case Study

Three studies carried out in Brunei Darussalam, by Khoo (2001), Lim (2000) and Noridah (1999) and supervised by the present writer have provided strong evidence that the traditional "teacher introduction-worked examples-seatwork" pattern in mathematics lessons is well entrenched within schools in that nation. To illustrate the extent to which the pattern manifests itself it will be useful to summarise Lim's (2000) study into the teaching and learning of algebra in four O-level classes at a secondary school. Altogether 94 students in four classes at a government secondary school ("School X") participated in the study. These classes were taught by three experienced, professionally trained, and mathematically well-qualified teachers. One teacher taught two of the classes.

Lim's research questions. Lim (2000) investigated four research questions:

- What knowledge and skills did the O-level students possess and how confident were the students about algebraic equations and factorisation in January 2000?

B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.) *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp. 203-210). Sydney: MERGA.

- What teaching and learning strategies did the teachers and students adopt in lessons on equations and factorisation between January 2000 and March 2000?
- What knowledge and skills concerned with algebraic equations and factorisation did the students learn between January 2000 and March 2000, and how did this affect their confidence to carry out related algebra tasks?
- What attitudes, societal forces and assumptions, had the greatest influence on the teaching and learning of algebraic equations and factorisation?

One of the classes (Form 4A) comprised 23 students who had gained high marks at the 1999 *Penilaian Menengah Bawah* PMB examination and were being prepared for the "Additional Mathematics" and the "Mathematics (Syllabus D)" examinations of the University of Cambridge Local Examination Syndicate (UCLES). The other 71 students, in Form 4B, Form 4C, and Form 4D, were preparing for UCLES's Mathematics (Syllabus D) examination, but *not* for the Additional Mathematics examination. All 94 students had passed both Mathematics and English at the 1999 PMB examinations.

The multiple perspectives research methodology. Lim (2000), like Noridah (1999) and Khoo (2001), employed a multiple perspectives methodology, with data being gathered from nine "vantage points". These vantage points were: (a) relevant documents; (b) pre- and post-teaching test performance; (c) responses by students and teachers to questionnaires; (d) classroom observations; (e) student confidence and other affective domain data; (f) students' study habits; (g) textbooks used by the students and teachers; (h) student artefacts; and (i) pre- and post-teaching interviews.

The Didactical Contract Identified by Lim (2000)

The climax of Lim's (2000) dissertation came in his identification of an unspoken, informal, but powerful "contract" which constrained the teaching and learning of mathematics in the fourclasses which he studied. Lim borrowed the idea of a "didactical contract" from Brousseau (see Balacheff, Cooper, & Sutherland, 1997), whose concept of a "didactical contract" embraces the obligations and sanctions that teachers and students impose on each other. It takes into account not only the knowledge that is to be taught and learned, but also the manner in which the teacher will teach and the students will behave.

When classroom observation data associated with the four O-level classes involved in Lim's (2000) study were triangulated with data from questionnaires and interviews, seven common elements and relationships within the didactical contracts between the three teachers and the students in the four classes were identified. Lim's (2000) analyses left no doubt that these didactical contracts were a vital component of the micro-cultures associated with the four mathematics classes he studied. The contracts greatly influenced not only what content was taught, but also how the teachers went about teaching that content and what the students did in their efforts to learn it.

The seven elements of the didactical contract identified by Lim (2000) were:

1. Class lessons should proceed in a formal manner, in which the teacher should teach students how to answer questions similar to those likely to appear on examinations. The teacher should not assume that the students know much about related work done in the past.
2. The teacher should briefly introduce a lesson by explaining the types of questions, which the class was about to consider. The teacher should then write (on the whiteboard) the solutions to a number of worked examples which

illustrate the main types of questions likely to be asked by examiners. After allowing the students sufficient time to copy the worked examples into their exercise books, the teacher should then set students to work on questions from past examination papers. After about 30 minutes of seatwork, during which time the teacher should move around the room providing assistance to students who might need it, the teacher should close the lesson by setting relevant examples for homework.

3. The teacher should not do anything which might embarrass a student publicly, and the students' behaviour should not embarrass the teacher. The teacher should not expect students to ask questions publicly. and the teacher should not ask individual students to answer difficult questions publicly. The only questions which teachers were "permitted" to ask were those at a sufficiently low cognitive level that chorused responses could be given.
4. The teacher should not give detailed mathematical explanations of difficult points, which would not be understood by most students. (Thus, for example, in lessons observed, the quadratic formula was merely stated, and not derived; its use was illustrated through worked examples.)
5. The teacher should not depart from the standard lesson sequence ("introduction to topic, followed by worked examples, followed by seatwork") except immediately before and after tests and examinations, when the teacher might spend the whole lesson going over examples with the class.
6. The students should copy, accurately, into their workbooks any points made on the whiteboard during the introduction, and also the worked examples written by the teacher on the whiteboard. Some homework (but not too much) should be expected of the students.
7. Provided the teacher behaved in the above-mentioned way, the students would co-operate. The teacher should permit the occasional lapse of concentration with students, and the students would tolerate the occasional "tantrum" from the teacher provided there was some justification for it.

The didactical contract outlined by Lim (2000) pointed towards a form of instruction which is similar to a pattern of instruction common in mathematics classes in the United States of America throughout the twentieth century (see, e.g., Cuban, 1993; Goodlad, 1984; Grossman & Stodolsky, 1994; Santagata & Stigler, 2000; Stigler & Hiebert, 1999). The same pattern has been found to be common in mathematics classrooms in Australian schools (see, e.g., Clarke, 1984; Mousley & Clements, 1990). Research indicates that this method of teaching rarely generates deep student understanding of concepts (Farrell & Farmer, 1988; Skemp, 1976; Suydam & Dessart, 1980). Indeed, in a landmark review of research, Resnick and Ford (1981) concluded that the practice of presenting mathematical concepts through stand-alone examples and repetitious practice sets does little to foster understanding or to affect transfer of learning to other areas.

Some Bruneian Performance Data

The Language Issue

Research by Clements (1999), Fatimah (1998), Sainah (1999) and Saman (2000) has shown that many upper-primary and lower secondary students in Brunei Darussalam understand very little of what they are taught in their mathematics lessons. Although that might be associated with the fact that mathematics is taught in *Bahasa Melayu* in Primary 1, 2, and 3, and in *English* in Primary 4 and above, it is salutary to note that upper primary and secondary students in Brunei Darussalam do no better on mathematics questions written in *Bahasa Melayu* than in English (Radiah, 1998; Saman, 2000).

Often the language transition issue is used as an excuse by teachers to concentrate solely on skill manipulation. Noridah (1999) reported that teachers tend to say that it is difficult to teach Bruneian students to solve mathematics word problems, and therefore they concentrate on teaching skills, because that will help their students pass examinations. Noridah (1999) found that A-level teachers who participated in her study consciously carried out time-cost-benefit analyses of how topics should be taught in order to maximise student performance. For some topics the teachers decided not to teach for "understanding" because that would take "too much time".

Many Bruneian students have difficulty understanding questions written in English. Consider, for example, the performance of Bruneian tenth-graders on the following question (taken from MacGregor (1991): "The number y is eight times the number z. Write this information in mathematical symbols". Only 11.7% of Form 4 O-level Mathematics students in Lim's (2000) study gave a correct response. 16.2% of the responses yielded the "reversal error" (i.e., $z = 8y$) and 71.2% of the responses were in the "other error" category. Responses in the "other error" category included: $y^8 = z$, $y^8 > z$, $8y > z$, y^8z, $y = z^8$, and $8yz$.

When Lim asked Form 4 teachers why students did so poorly on a question which should not be expected to be difficult for even Form 1 students, the teachers said students tended to forget everything they learned in mathematics classes in earlier years. Data put forward by Khoo (2001), Lim (2000), Noridah (1999), Radiah (1998), Sainah (1999) and Saman (2000) suggest that the teachers' explanation was probably correct. Khoo (2001) showed that most of the O-level students in her Form 5 sample could not answer correctly geometry questions which had appeared on the Form 3 PMB examination.

Some Further Algebra Performance Data

Lim (2000), Noridah (1999), and Radiah (1998) found that most O-level students who had "passed" the end-of-Form 3 PMB national mathematics examination knew little about the algebra taught to them in Forms 1, 2 and 3. When Lim (2000) asked his four O-level classes to attempt 30 algebra questions based on lower secondary algebra, the results were as shown in Table 1. Although almost all of the 30 questions involved only elementary factorisation and equations covered in Forms 1 through 3, the mean number of correct answers given by the students in Form 4B, 4C and 4D was about 2.6. All of the students in those classes had passed the 1999 PMB Mathematics examination.

Table 1
Performance on 30 Algebra Questions by Four Form 4 O-Level Classes (Lim, 2000)

Form	No. of Students	Mean Score/30	Standard Deviation	Lowest Score/30	Highest Score/30
4A	23	6.7	3.4	2	14
4B	22	4.3	3.2	0	13
4C	20	1.6	1.2	0	5
4D	29	1.9	1.9	0	7
Overall	94	3.6	3.2	0	14

Table 2 shows performances, by the different classes, on selected questions. The national mathematics curriculum specifically includes factorisation of the difference of two squares (i.e., $a^2x^2 - b^2y^2$), in the Form 2 syllabus and questions on the difference of two squares are included each year on the PMB mathematics examinations. It is a matter of surprise then, that so few students factorised $x^2 - 4z^2$ correctly. Even the results of students in the top-stream Form 4A were not uniformly good. For example, when asked to solve the equation $x^2 = 9$, no student in Form 4A gave the answer "$x = 3$ or $x = -3$" (or "$x = \pm 3$").

Table 2
Class Comparisons for Questions on Factorisation (from Lim, 2000)

	Percentage Correct for Factorising ...			
Form	Q. 19 $2x - 4$	Q. 20 $4xy^2 - 9xy$	Q. 22 $x^2 - 4x + 4$	Q. 25 $x^2 - 4z^2$
4A	65.2%	30.4%	17.4%	26.1%
4B	40.9%	22.7%	4.5%	22.7%
4C	10.0%	5.0%	0.0%	5.0%
4D	10.3%	3.4%	0.0%	10.3%
Overall	30.9%	14.9%	5.3%	16.0%

After the four Form 4 classes had been given lessons specifically on factorisation and solving equations, the means (out of 30) for the classes on the 30 algebra questions became 11.7 (Form 4A), 10.1 (Form 4B), 5.3 (Form 4C) and 3.0 (Form 4D). Although pre-teaching/post-teaching class mean gains were highly statistically significant, that is misleading because these "top" Form 4 students still were unable to do most of the standard skill-based questions which they had just been taught to do. In fact, post-teaching interviews indicated that students had learned little from the drill and practice routines. Even students who could factorise simple algebraic expressions merely followed rules, and did not know why the manipulations they carried out actually worked. The best students could factorise, but hardly any of them knew what the word "factor" meant in the context of algebra. Likewise, students could solve simple equations but did not know what the word "solution" meant in the context of algebraic equations.

Interview Data

This fundamental lack of understanding is revealed in the following excerpt of an interview with a Form 4 O-level mathematics student (taken from Lim, 2000) which took place after the student had attended O-level lessons in which factorisation and equations had been specifically taught. The interview centred around the Form 4 student's efforts to factorise $2x - 4$, a skill which had been taught in Form 2, and again in Forms 3 and 4.

Immediately before the interview the student generated a three-line response to a question asking her to "factorise completely $2x - 4$." She had obtained the answer "$x = 6$." During the interview the following "conversation" occurred (I = Interviewer, S = Student):

Interviewer: Let's try Question 19 please *[requiring the student to factorise completely, $2x - 4$].*
Student: *[Student tried.]*
Interviewer: Can you please read the instruction?
Student: Factorise each of the following expressions completely. All working must be shown.
Interviewer: Can you read Question 19 please?
Student: $2x$ minus 4.
Interviewer: So, what does the question want you to do?
Student: To factorise.

Interviewer:	What does it mean?
Student:	To make the number smaller.
Interviewer:	Can you explain what you have done?
Student:	$2x$ minus 4 equals to x equals to 4 plus 2. x equals to 6.
Interviewer:	Can you explain how you get x equals 4 plus 2?
Student:	Transfer 2 to the other side. Then I get x equals to 4 plus 2.
Interviewer:	How do you get 4 here *[pointing at $x = 4 + 2$]*?
Student:	I put 4 here and then minus 4 change to plus 4.
Interviewer:	OK. What is your answer?
Student:	x equals to 6.
Interviewer:	What has $x = 6$ got to do with the original question?
Student:	I don't know.
Interviewer:	How do you check your answer?
Student:	Ask the teacher for the answer.

Major Factors Influencing the Poor Mathematics Performance and Understanding of Many Bruneian Students

Outside-of-Class Time Spent Studying Mathematics

Homework and outside-of school tuition in mathematics. Data generated by Lim's (2000) *Study Habits Questionnaire* revealed that 64 percent of the students involved in the study reported spending not more than 1 hour per week doing mathematics homework. In May 2000, almost all of the students stated that they had not taken extra tuition classes in mathematics that year; half of them indicated they would not take private tuition in mathematics later in the year. In Japan and Singapore, by contrast, almost all secondary students receive supplementary tuition in mathematics (Menon, 2000; Schümer, 1999).

When the students were asked whether they used textbooks other than those provided by the school for mathematics homework, most replied "no". Students said that they got most of their mathematics homework questions from the book which they took to class (i.e., Dyna Publisher, 1999)—which, in fact, was a compilation of past examination papers.

The drill-and-practice, examination-orientation of the lessons on algebra. Khoo's (2001), Lim's (2000) and Noridah's (1999) analyses suggest that in the Bruneian context the strong emphasis on drill and practice, and the strong examination orientation in lessons, actually *causes* poor performance and poor understanding. The students did not learn mathematics relationally, and in test and examination situations they struggled to work out which mathematics was needed for which problems. Many of the students had committed a large number of mathematical facts and skill procedures to memory, but because the volume of the memorised material was so great they often forgot parts of it.

Summary and Concluding Comments

Study habits data reported by Khoo (2001) and Lim (2000) indicated that on average the O-level students (at different schools) spent only one hour per week on mathematics homework. That is well below the amount of time spent on mathematics homework by students in Japan, Singapore, the United States and Germany (Menon, 2000; Schümer, 1999). By contrast with the situation in Japan and Singapore, few Bruneian students obtained private tuition in mathematics, and most did not obtain "quality" help at home.

The teachers in Lim's study were resigned to the likelihood that, with the exception of the top-stream Form 4A students, most students would obtain low grades on the O-level examinations. From the teachers' perspective that was realistic because most O-level classes at the school throughout the 1990s had obtained low grades for Mathematics.

Perhaps cramming for examinations, inherent in the didactical contract identified by Lim, was the root of the performance problem. The drill and practice routines, and low-level use of language, led students to regard mathematical "understanding" as something equivalent to being able to answer examination questions correctly. They had come to believe that the only way to learn mathematics well was through drill and practice.

Data reported in Noridah's (1999), Khoo's (2001) and Lim's (2000) studies indicate that in pressure situations (e.g., in tests and formal examinations) most of the students could not remember which skills should be associated with which problems. That was probably because the students had tended to try to learn a whole host of separate skills by rote, and their short-term memories had been strained to the point of bursting.

The lessons analysed by Khoo (2001), Lim (2000) and Noridah (1999) tended to be very examination-oriented. In each lesson a teacher-led introduction was followed by worked examples (which the students copied), and then seatwork, in which students attempted questions from past examination papers. There appeared to be a powerful didactical contract operating, which defined how teachers and students should behave. The main element of the agreement was that the teacher should provide the students with notes and worked examples, and students should do their best to answer questions from past examination papers. The teacher should not expect too much of students in the classroom, and not too much pressure should be put on them to do homework.

In Lim's (2000) and Khoo's (2001) studies, after the lessons on algebraic equations and factorisation (Lim) and the circle theorems (Khoo), the class means of the students' performances on relevant tests were only slightly higher than at the pre-teaching stage. Even after the lessons, many of the students still knew very little about the topics concerned. In most cases, any learning which had occurred was almost entirely of the instrumental kind, with students struggling to remember the rules they had been taught.

The teachers in Khoo's, Lim's, and Noridah's studies were aware of their students' low levels of understanding of basic mathematical concepts, and their students' imperfect knowledge of associated mathematical skills. The teachers blamed that reality on the fact that the students had failed to learn mathematics satisfactorily in Primary 6 and in Forms 1 through 3. In other words, they thought the students had moved on to upper secondary mathematics without possessing the fundamental mathematical skills they would need at that level. All the teachers agreed that a pass on the end-of-Form 3 PMB mathematics examinations did not guarantee possession of basic mathematical knowledge and skills. They acknowledged that students prepared for examinations by memorising formulae. As soon as the examinations were over the students were happy to forget everything.

O-level teachers felt pressured to move through the O-level syllabus as quickly as possible, to leave plenty of time for preparing students for tests and examinations (mainly by going through past examination papers). From the teachers' perspective, there was no time to teach for relational understanding. Examinations waited for no one.

An important part of the didactical contract was the use of questions from past examination papers in class, and for homework. For students, that guaranteed they were being well prepared for examinations, especially the high-stakes examination. However, this paper has raised the fundamental issue of what "well prepared" means in that context.

References

Balacheff, M., Cooper, M., & Sutherland, R. (1997). *Theory of didactical situations in mathematics (Didactique des mathèmatiques—Guy Brousseau).* Dordrecht: Kluwer Academic Publishers.

Clarke, D. J. (1984). Secondary mathematics teaching: Towards a critical appraisal of current practice. *Vinculum, 21*(4), 16–21.

Clements, M. A. (1999). Language aspects of mathematical modelling in primary schools. In M. A. Clements & Y. P. Leong (Eds.), *Cultural and language aspects of science, mathematics, and technical education* (pp. 363–372). Gadong: Universiti Brunei Darussalam.

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms 1890-1990* (2nd ed.). New York: Teachers College Press.

Dyna Publisher (1999). *Classified questions (1981–1998) Mathematics (Syllabus D), G.C.E. O-level June/November.* Singapore: Author.

Farrell, M. A., & Farmer, W. A. (1988). *Secondary mathematics instruction: An integrated approach.* Providence, R.I.: Janson Publications.

Pg Hj Fatimah (1998). *Assessing misunderstanding in fractions by low achieving primary six pupils.* Unpublished M.Ed field project, Universiti Brunei Darussalam.

Goodlad, J. I. (1984). *A place called school.* New York: McGraw Hill.

Grossman, P. L., & Stodolsky, S. S. (1994). Considerations of content and the circumstances of secondary school teaching. In L. Darling-Hammond (Ed.), *Review of Research in Education 20* (pp. 179-221). Washington, D.C.: American Educational Research Association.

Khoo Siew Chin (2001). *The teaching and learning of geometry.* Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

Lim Ting Hing (2000). *The teaching and learning of algebraic equations and factorisation in O-level Mathematics: A case study.* Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

MacGregor, M. (1991). *Making sense of algebra: Cognitive processes influencing comprehension.* Geelong, Victoria: Deakin University.

Mousley, J. A., & Clements, M. A. (1990). The culture of mathematics classrooms. In K. Clements (Ed.), *Whither mathematics?* (pp. 387–406). Melbourne: Mathematical Association of Victoria.

Menon, R. (2000). Should the United States emulate Singapore's education system to achieve Singapore's success in the TIMSS? *Mathematics Teaching in the Middle School, 5*(6), 345–347.

Hjh Noridah bte Abdullah (1999). *The teaching and learning of inverse functions.* Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

Hjh Radiah bte Hj Mohidin (1998). *The difficulties faced by students of Brunei Darussalam in transforming short mathematical word statements into algebraic form.* Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction.* Hillsdale, NJ: Lawrence Erlbaum.

Hjh Sainah binti Hj Nayan (1998). *Problem-solving errors by Primary six children in specialist teachers' project schools.* Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

Hj Saman bin Hj Anit (2000). *Investigating understanding by Primary Six pupils of word problems involving multiplication and division.* Unpublished M.Ed project, Universiti Brunei Darussalam.

Santagata, R., & Stigler, J. W. (2000). Teaching mathematics: Italian lessons from a cross-cultural perspective. *Mathematical Thinking and Learning, 2*(3), 191–208.

Schümer, G. (1999). Mathematics education in Japan. *Journal of Curriculum Studies, 31*(4), 399-427.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77,* 20–26.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.

Suydam, M. N., & Dessart, D. J. (1980). Skill learning. In R. J. Shumway (Ed.), *Research in mathematics education* (pp. 207–243). Reston, VA: National Council of Teachers of Mathematics.